2. Form Factor Programme: Some References

2. The origin of the form factor programme is generally attributed to Karowski & Weiss:

   P. Weisz, Phys. Lett. B 67 (1977) 179.
   M. Karowski, P. Weisz, Nucl. Phys. B 139 (1978) 445.

   who studied the form factors of the sine-Gordon model. However, the programme experienced much development thanks to the work of Smirnov from where many of the ideas and techniques discussed in the lecture originate. In particular, he introduced the kinematic and bound state residue equations and provided the first examples of their systematic solution. Some of this extensive work was then collected in the book:

   F.A. Smirnov, Form factors in Completely Integrable Models of Quantum Field Theory, Adv. Series Math. Phys., Vol. 14, World Scientific, Singapore, 1992.

   The version of the equations which includes a factor of local commutativity ω was first used in:

   V.P. Yurov and Al. B. Zamolodchikov, Int. J. Mod. Phys. A6 (1991) 3419.

   In order to study the form factors of the Ising model. In this case, to account for both operators σ and μ a factor of local commutativity $\omega = -1$ must be introduced for the operator μ (otherwise the kinematic residue equation gives zero residue).

   A nice feature of this programme is that for some theories it has been found that the form factors may be expressed as determinants of matrices whose entries are elementary symmetric polynomials. These determinant formulae (once guessed from the first few cases) can then be proven in generality. Nice examples of this can be found in:

   Al. B. Zamolodchikov, Nucl. Phys. B348 (1991) 619.

   for the Lee-Yang model, the simplest example for a theory with a bound state pole.

   A. Fring, G. Mussardo and P. Simonetti, Nucl. Phys. B393 (1993) 413; Phys. Lett. B307 (1993) 83.
   A. Koubek and G. Mussardo, Phys. Lett. B311 (1993) 193.

   for the sinh-Gordon model, the simplest interacting integrable model, consisting of a single particle spectrum and no bound states.

O. A. Castro-Alvaredo, A. Fring and C. Korff, Phys. Lett. B484 (2000) 167.
O. A. Castro-Alvaredo and A. Fring, Nucl. Phys. B604 (2001) 367?390.
O. A. Castro-Alvaredo and A. Fring, Phys. Rev. D64 (2001) 085007.

for the more involved homogeneous sine-Gordon models whose spectrum contains several particles and whose operator content is very involved.

As mentioned in the lecture, one of the challenges when computing form factors is the operator identification. There have been numerous works which have considered this question seriously. One of the most complete and beautiful results is due to Cardy & Mussardo who studied the Ising model form factors and were able to show a one-to-one correspondence between the number of fields of a certain spin and the number of states in the Verma module of the CFT for the same spin:

J. L. Cardy and G. Mussardo, Nucl. Phys. B 340 (1990) 387.

The question of operator identification was also addressed in:

A. Koubek and G. Mussardo, Phys. Lett. B311 (1993) 193.
O. A. Castro-Alvaredo and A. Fring, Nucl. Phys. B604 (2001) 367?390.
G. Delfino and G. Niccoli, JHEP 0605 (2006) 035.
G. Delfino and G. Niccoli, Nucl. Phys. B799 (2008) 364-378.

The form factor programme has been generalised in various ways. One generalisation is concerned with the study of theories in the presence of a boundary. The boundary form factor programme was proposed in:

Z. Bajnok, L. Palla and G. Takács, Nucl. Phys. B750 (2006) 179-212.

And a nice example of its use for the exponential fields of boundary sinh-Gordon can be found in:

G. Takacs, Nucl. Phys. B 801 (2008) 187-206,

where the cumulant expansion is used for a theory with a boundary.

The form factor programme has also been generalised to deal with fields which have more complicated semi-locality properties, like the branch point twist field that arises in the context of entanglement:

J. L. Cardy. O. A. Castro-Alvaredo and B. Doyon, J. Stat. Phys. 130 (2008) 129?168.

In the lecture it was mentioned that there is a bound to the maximum exponential divergency of the form factors as any of the rapidities tends to infinity. This bound was first presented in:

G. Delfino and G. Mussardo, Nucl. Phys. B455 (1995) 724.

and it follows from considering the two-point function of a particular field in its euclidean Lehmann representation.

We have also seen the cluster decomposition property in momentum space. An argument that relates this property to chiral factorization in CFT was given in:

G. Delfino, P. Simonetti and J.L. Cardy, Phys. Lett. B 387 (1996) 327.

Clustering in momentum space has been observed for many theories in works such as:

F. Smirnov, Nucl. Phys. B337 (1990) 156.
Al. B. Zamolodchikov, Nucl. Phys. B348 (1991) 619.
A. Koubek and G. Mussardo, Phys. Lett. B311 (1993) 193.
G. Mussardo and P. Simonetti, Int. J. Mod. Phys. A9 (1994) 3307.

In:

O. A. Castro-Alvaredo and A. Fring, Nucl. Phys. B604 (2001) 367?390.

it was shown that clustering could be employed to obtain the form factors of 10 distinct operators from the the form factors of a single field.

In:

H. Babujian and M. Karowski, Phys. Lett. B471 (1999) 53.

clustering was proven for the free massive Boson based on Weinberg's counting argument.

The c-theorem was proven in:

A. B. Zamolodchikov, JETP Lett. 43 (1986) 730.

and a more detailed re-derivation can also be found in:

P. Gingsparg, Applied Conformal Field Theory, in Fields, Strings and Critical Phenomena, ed. E. Brèzin and J. Zinn-Justin, Les Houches 1988, Session XLIX, North-Holand, Amsterdan 1990 (p. 169).
O. A. Castro-Alvaredo, PhD Thesis (2000), hep-th/0109212 (p. 131).

Since then, the c-theorem has been extensively used as a consistency check for the form factors of the stress-energy tensor. See e.g.

A. Fring, G. Mussardo and P. Simonetti, Nucl. Phys. B393 (1993) 413.
G. Mussardo and P. Simonetti, Int. J. Mod. Phys. A9 (1994) 3307.

However, it has mostly been used to obtain the central charge, rather than the full c-function. A nice study of the c-function for a complicated interacting theory can be found in:

O. A. Castro-Alvaredo and A. Fring, Phis. Rev. D63 (2000) 021701(R).

The Δ-sum rule was proposed in:

G. Delfino, P. Simonetti and J.L. Cardy, Phys. Lett. B 387 (1996) 327.

And has since then been used for many models in order to identify the underlying conformal dimension of particular fields. For example, it is used in the original paper to obtain the conformal dimension of the field μ in the Ising model. An interesting feature is that, like the c-theorem, the Δ-sum rule can be also used to obtain a function $\Delta(r)$ which flows between the IR and the UV and which is not necessarily monotonic. The following paper provides a nice example of this:

O. A. Castro-Alvaredo and A. Fring, Phis. Rev. D63 (2000) 021701(R).