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TWO-POINT CORRELATION FUNCTION IN SCALING LEE–YANG MODEL

Al.B. ZAMOLODCHIKOV

*International School for Advanced Studies, Trieste, Italy**

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The structure of the UV singularity in the two-point correlation function is considered f the scaling Lee–Yang model. Both perturbative and nonperturbative corrections to UV confc mal theory are discussed. The IR convergent perturbation theory for the structure functions operator algebra is developed and the first-order corrections are calculated explicitly. The U expansion is compared numerically with the results of partial summation over intermedia asymptotic states. These two expansions match well in the intermediate region and give reasonable precision data for the correlation function in the whole region of scaling distance

1. Introduction

The conformal field theory (CFT) [1–3], being the theory of renormalizatio group fixed point, provides us (among other applications) with the classification c possible ultraviolet (UV) behavior in general relativistic field theory (RFT). Be cause of extremely high symmetry, CFT models are typically exactly solvable an today we have an enormous number of explicit constructions (see e.g. refs. [1–11] From this point of view in approach to general RFT it is natural to begin with th short-distance CFT and consider the corresponding renormalization group trajec tory as a perturbation of CFT model by a suitable relevant (or marginal) scala operator (see e.g. refs. [12–20], where this approach was applied to different 2D problems). As a starting point one usually takes the conventional action

$$A_{\mathrm{RFT}} = A_{\mathrm{CFT}} + g \int \varphi(x)\, \mathrm{d}^2 x, \qquad (1.1)$$

where the scalar CFT field $\varphi(x)$ has dimension $\Delta(=\bar{\Delta}) \leqslant 1$. The coupling constant g develops positive scale dimension $g \sim (\mathrm{mass})^{2-2\Delta}$ for $\Delta < 1$ and becomes dimensionless in the marginal case $\Delta = 1$. The last case must correspond to asymptotically free renormalization group behavior to make sense in the picture under consideration.

In any case in the UV limit the coupling constant becomes small. It is therefore natural to expect that perturbation theory in g works well in the UV limit, providing us with a systematic short-distance expansion of observables. Consider, for example, the perturbative expansion of a particular RFT correlation function, say the two-point one,

$$\langle \Phi(x)\Phi(0)\rangle_{\mathrm{RFT}}, \tag{1.2}$$

where Φ is some local field. For simplicity it is supposed to be scalar. One may try the following formal expansion:

$$\langle \Phi(x)\Phi(0)\rangle_{\mathrm{RFT}} = \sum_{n=0}^{\infty} \frac{(-g)^n}{n!} \int \langle \tilde{\Phi}(x)\tilde{\Phi}(0)\varphi(y_1)\ldots\varphi(y_n)\rangle_{\mathrm{CFT}}\, \mathrm{d}^2 y_1 \ldots \mathrm{d}^2 y_n, \tag{1.3}$$

where the correlation functions in the right-hand side are calculated in CFT and $\tilde{\Phi}(x)$ is the corresponding UV limit of the field $\Phi(x)$. However, when attempting to calculate the integrals in the r.h.s. of (1.3), one readily encounters both infrared (IR) and UV divergencies. The latter can be handled by standard renormalization techniques and lead to renormalizations of local fields and, in the marginal case $\Delta = 1$, of the coupling constant. The marginal situation seems much more complicated for analyses and in what follows we only consider strictly relevant perturbations $\Delta < 1$. Moreover we suppose that $\Delta < 1/2$ to avoid completely any renormalizations in the interaction hamiltonian.

As for the IR divergencies, they cannot be absorbed into any local entities of the theory and lead to known non-analyticity in the coupling constant. In this paper we try to handle these nonperturbative corrections. The point of view is the following. To estimate UV behavior of e.g. the two-point function (1.2), we start with the local operator product expansion (OPE)

$$\Phi(x)\Phi(0) = \sum_i C^i_{\Phi\Phi}(x)\, A_i(0), \tag{1.4}$$

where $A_i(0)$, $i = 0, 1, \ldots$ is the complete set of local fields in the theory and $C^i_{\Phi\Phi}(x)$ are the corresponding structure functions. It is natural to expect that these quantities, being local, do not develop any non-analyticity and have regular expansions in the coupling constant g. More strictly we suppose that the basis $A_i(0)$, $i = 0, 1, \ldots$ in the space of local fields $\mathscr{A}$ can always be chosen so that all the structure functions are analytic in g.

The basic fields $A_i(0)$ are considered as perturbations of the corresponding scale covariant CFT fields $\tilde{A}_i(0)$. Denoting the dimensions of these CFT fields as $(\Delta_i, \bar{\Delta}_i)$, we have from dimensional arguments

$$C^i_{\Phi\Phi}(x) = x^{\Delta_i - 2\Delta_\Phi}\bar{x}^{\bar{\Delta}_i - 2\bar{\Delta}_\Phi} \sum_{n=0}^{\infty} C^{i(n)}_{\Phi\Phi}\left(g r^{2-2\Delta}\right)^n, \tag{1.5}$$

where $r = (x\bar{x})^{1/2}$ is the scalar distance. The price we have to pay for the analyticity of the structure functions is nonzero vacuum expectation values (VEVs) of some of the fields A_i. These values are of course of nonlocal nature and may be non-analytic. Indeed, again for dimensional reasons,

$$\langle A_i\rangle = g^{\Delta_i/(1-\Delta)} Q_i, \tag{1.6}$$

where the Q_i are dimensionless numbers. The set of VEVs $\langle A_i\rangle$, $i = 0, 1, \ldots$ is a characteristic of large-distance environment of our local OPE world. We call it the outvacuum (outvac) vector (as opposed to local invacuum (invac) vector I, i.e. the identity operator). All the non-analyticity is absorbed into the outvac vector.

Of course, the majority of the VEVs $\langle A_i\rangle$ is zero due to RFT symmetries. First, the VEVs of the fields with nonzero spin vanish. Secondly, all the fields which are spatial derivatives of other ones do the same. And at last many RFT models exhibit additional symmetries generated by an infinite set of commuting integrals of motion [14, 15, 17–24]. These are so-called integrable models. In this case every field, which can be generated by action of any integral of motion, cannot develop nonzero VEV. Finally we are left with the factor space $\mathscr{P}_0$, spanned by "primary" (with respect to spatial differentiation and the higher symmetries) scalar fields A_ν, $\nu = 0, 1, \ldots$.

Selecting these "primary" fields in the r.h.s. of eq. (1.4), we obtain

$$\langle \Phi(x)\Phi(0)\rangle_{\mathrm{RFT}} = \sum_\nu C^\nu_{\Phi\Phi}(x)\langle A_\nu\rangle. \tag{1.7}$$

In this paper we try this construction for a presumably simplest interacting RFT – the scaling Lee–Yang model (SLYM) [25]. From the UV point of view SLYM corresponds to unique perturbation of the minimal CFT model $\mathscr{M}(2/5)$ [19, 26]. As a test laboratory the example chosen has many advantages.

(i) The corresponding CFT space of states is very simple and includes only two primary fields [1, 26]. These are the identity operator I and the scalar field φ of negative dimension $\Delta = -1/5$. SLYM corresponds to perturbation of $\mathscr{M}(2/5)$ exactly by this operator. Therefore it is safe to keep for this field the same notation as for the perturbation field in eq. (1.1). The coupling constant is purely imaginary,

$$A_{\mathrm{SLYM}} = A_{\mathscr{M}(2/5)} + ih\int \varphi(x)\, \mathrm{d}^2 x, \tag{1.8}$$

where $h \sim (\text{mass})^{12/5}$.

(ii) As it was demonstrated in ref. [19], SLYM is integrable and massive RFT. Its on-mass-shell spectrum consists of a single massive neutral particle A of mass m. The scattering theory is factorized with the following two-particle amplitude [19]:

$$S_{AA}(\beta) = \frac{\sinh\beta + i\sin\frac{1}{3}\pi}{\sinh\beta - i\sin\frac{1}{3}\pi} \; = \frac{\langle 1\rangle\langle 2\rangle}{\langle -1\rangle\langle -2\rangle} \tag{1.9}$$

The pole at $\beta = 2i\pi/3$ in $S_{AA}(\beta)$ is interpreted as a bound state corresponding to fusion, $AA \to A \to AA$. The wrong sign in the residue signals the absence of unitarity in the SLYM scattering theory.

(iii) Additional information is provided by the thermodynamic Bethe ansatz (TBA) approach [27]. What will be important for us is the VEV of the stress tensor trace $\Theta = \frac{1}{4}T^{\mu}_{\mu}$,

$$\langle\Theta\rangle = -\pi m^2/4\sqrt{3}\,. \tag{1.10}$$

(iv) In what follows we also use the relation between the coupling h and the mass scale m of the theory,

$$h = 0.09704845636\ldots \times m^{12/5}, \tag{1.11}$$

found in ref. [27] by numerical integration of the thermodynamic Yang–Yang equation for SLYM.

In sect. 2 the above general arguments are specified for the two-point correlation $G(r)$ of the stress tensor trace $\Theta(x)$ in SLYM. Using a zero-order approximation for the relevant structure functions, we are able to develop a UV expansion up to order $o(r^{14/5})$. In this calculation the exact VEV of Θ is used essentially.

In sect. 3 the IR-convergent perturbation theory for the OPE structure functions is formulated and the first-order corrections are evaluated. This gives an estimation of the correlation function $G(r)$ up to order $o(r^{24/5})$. Further development of the perturbation theory makes no sense. At the order $O(r^{24/5})$ the next "primary" operator (namely $:\bar{T}T:$) begins to contribute to the correlation function. Up to now I hardly know how to find this VEV (and also VEVs of higher "primary" operators) exactly.

Integrability of the model allows one to compare the obtained UV expansion with an alternative set of data. Inserting a complete set of asymptotic states between two operators in the correlation function $\langle\Theta(x)\Theta(0)\rangle$, one obtains an opposite, large-distance expansion in the number of intermediate particles. To calculate the expansion terms one needs matrix elements of the operator $\Theta(x)$ between asymptotic states (form factors). A systematic way of recovering these quantities in integrable theories from the factorized scattering data was developed in refs. [28–37]. In ref. [38] it was proposed to use the exact form factors to study numerically correlation functions in integrable RFT models. In sect. 4 the scattering theory (1.9) is used to construct the whole set of $\Theta(x)$ form factors. These form factors were obtained also from the sine-Gordon ones in ref. [37], where it was shown that SLYM can be constructed by eliminating solitons from the asymptotic space of the sine-Gordon model with special coupling.

Using the exact form factors we calculate numerically zero-, one-, two- and three-particle contributions to the correlation function $G(r)$. This leads to a set of data precise enough up to $r \sim 0.01\,m^{-1}$. Direct comparison of the data provided by the short- and large-distance expansions is possible due to relation (1.11). It shows that these two sets of data match well in the intermediate region $0.01\,m^{-1} < r < 1.00\,m^{-1}$, providing us with a reasonable precision numerical estimation of $G(r)$ in the whole region of scaling distances mr.

2. Two-point correlation in SLYM; short-distance expansion

The simplest nontrivial correlation in SLYM [apart from the VEV (1.10)] is that of two CFT primary fields $\varphi(x)$. We suggest here that the local fields in the perturbed theory are in one-to-one correspondence with that of the short-distance CFT [18]. In order not to invent new designations we keep the CFT notations for them. It will be seen below that in perturbation theory these fields in general differ from their CFT counterparts in infinite additive renormalizations.

We start with the unperturbed CFT $\mathscr{M}(2/5)$, where OPE of two fields $\varphi(x)$ has the form

$$\varphi(x)\varphi(0) = r^{4/5}\left(I + \frac{r^4}{121}:\bar{T}T:(0) + O(r^{12})\right) + \mathscr{C}_{\varphi\varphi\varphi}r^{2/5}\left(\varphi(0) + \frac{r^8}{3249}F(0) + O(r^{12})\right). \tag{2.1}$$

Here $r = (x\bar{x})^{1/2}$ and in the r.h.s. we only keep scalar nonderivative (conformal) operators. The conformal field $F(0)$ is defined as

$$F(0) = \left(L_{-4} - \tfrac{625}{624}L^4_{-1}\right)\left(\bar{L}_{-4} - \tfrac{625}{624}\bar{L}^4_{-1}\right)\varphi(0)\,. \tag{2.2}$$

The structure constant $\mathscr{C}_{\varphi\varphi\varphi}$ in $\mathscr{M}(2/5)$ was calculated in refs. [4,5,26]. It is purely imaginary and we denote $\mathscr{C}_{\varphi\varphi\varphi} = i\kappa$ to deal with the real number

$$\kappa = \tfrac{1}{2}\gamma^{3/2}\left(\tfrac{1}{5}\right)\gamma^{1/2}\left(\tfrac{2}{5}\right) = 1.911312699\ldots, \tag{2.3}$$

where the abbreviation $\gamma(x) = \Gamma(x)/\Gamma(1-x)$ was used.

The corresponding OPE in SLYM differs from (2.1) in structure functions, ıich are no more simple powers,

$$\varphi(x)\varphi(0) = C^{I}_{\varphi\varphi}(r)I + C^{\varphi}_{\varphi\varphi}(r)\varphi(0) + C^{\bar{T}T}_{\varphi\varphi}(r):\bar{T}T:(0) + \dots \quad (2.4)$$

ıving in mind to take VEV we again omit here all derivatives and operators with ınzero spin.

Following the line suggested above we suppose regular perturbative expansions r the structure functions,

$$C^{I}_{\varphi\varphi\varphi}(r) = r^{4/5}(1 + Q^{I}_{1}t + Q^{I}_{2}t^{2} + \dots),$$

$h \sim m^{2-2\Delta}$

$$C^{\varphi}_{\varphi\varphi\varphi}(r) = C_{\varphi\varphi\varphi}r^{2/5}(1 + Q^{\varphi}_{1}t + Q^{\varphi}_{2}t^{2} + \dots),$$

$(mr)^{12/5}$

$$C^{\bar{T}T}_{\varphi\varphi}(r) = \tfrac{1}{121}r^{24/5}(1 + Q^{\bar{T}T}_{1}t + \dots), \quad (2.5)$$

ıere t is the dimensionless coupling constant $t = hr^{12/5}$. The next term omitted in |. (2.4) is the renormalized version of the CFT operator (2.2). It is of order $r^{42/5}$. The expansions in (2.5) are supposed to be convergent in a finite region ound $t = 0$.

The field $\varphi(x)$ is purely imaginary. It is therefore more convenient to use stead the stress tensor trace $\Theta(x)$, which is related to $\varphi(x)$ as follows:

$$\Theta(x) = ih\pi(1-\Delta)\varphi(x), \quad (2.6)$$

here $\Delta = -1/5$ is the dimension of the field φ. For the corresponding two-point ınction

$$G(r) = \langle\Theta(x)\Theta(0)\rangle \quad (2.7)$$

e therefore find

$$G(r) = -\pi^2(1-\Delta)^2h^2C^{I}_{\varphi\varphi}(r) + i\pi(1-\Delta)hC^{\varphi}_{\varphi\varphi}(r)\langle\Theta\rangle$$
$$-\pi^2(1-\Delta)^2h^2C^{\bar{T}T}_{\varphi\varphi}(r)\langle:\bar{T}T:\rangle + \dots \quad (2.8)$$

'he outvac is supposed to be normalized so that $\langle I\rangle = 1$. Substituting for $\langle\Theta\rangle$ the alue (1.10) and zero-order terms for the structure functions $C^{I}_{\varphi\varphi}(r)$ and $C^{\varphi}_{\varphi\varphi}(r)$, we ind up to order $r^{14/5}$,

$$G(r) = -\pi^2(1-\Delta)^2h^2r^{4/5} + \frac{\pi^2(1-\Delta)h\kappa m^2}{4\sqrt{3}}r^{2/5} + O(r^{14/5}). \quad (2.9)$$

$2-2\Delta$

$2-2\Delta = 2+\frac{2}{5} = 12/5$

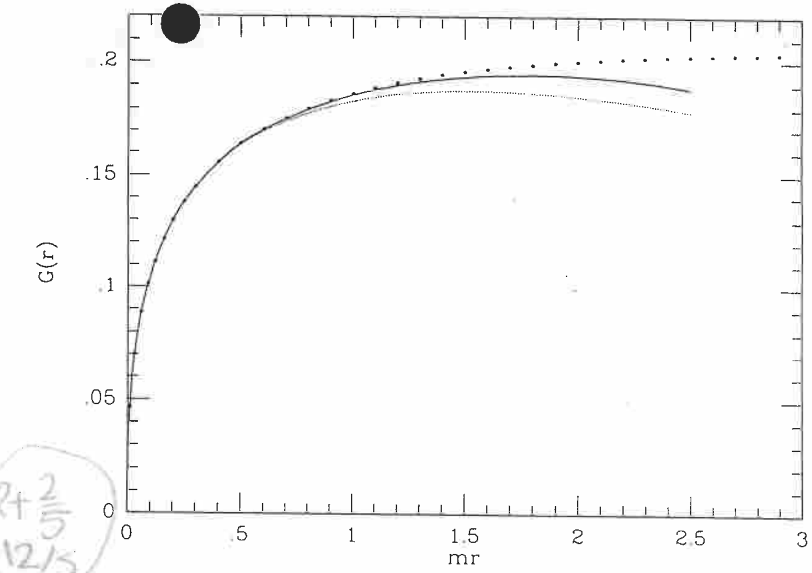

Fig. 1. Two-point function G (in units m^4) versus scaling distance mr. Dotted line: zero-order short-distance expansion (2.9). Full line: the same corrected by first-order perturbative terms in structure functions [eqs. (3.22) and (3.27)]. Full circles: large-distance expansion with up to three-particle contributions included.

It turns out that due to nonzero VEV of the field $\varphi(x)$ the two-point correlation function $G(r)$ behaves for small r not as $r^{4/5}$, as one could expect naively from CFT arguments, but rather as $r^{2/5}$. The competition of the two terms quoted in eq. (2.9) results in the curve drawn in fig. 1 in the "physical" mass scale, i.e. in units of m^{-1}. The relation (1.11) was used for this rescaling.

The next subleading terms of orders $r^{14/5}$ and $r^{16/5}$ come from the first-order perturbative corrections to the structure functions $C^{I}_{\varphi\varphi}(r)$ and $C^{\varphi}_{\varphi\varphi}(r)$. These corrections are calculated in sect. 3 where the IR-convergent perturbation theory for structure functions is settled.

To obtain further we need the VEV of the next field $:\bar{T}T:$. Now it is not clear, however, how this value can be calculated exactly. In any case it is seen in fig. 1 that the zero-order terms we have taken into account in eq. (2.9) already give a reasonable match with the large-distance expansion data (see sect. 4 below).

3. Perturbation theory for structure functions

The OPE structure functions are purely local characteristics of RFT and therefore do not carry any information on large-distance environment. In particular they know nothing about the situation near spatial infinity and cannot suffer from IR divergencies. In fact in perturbation theory all the integrals contributing to the structure functions can be restricted inside a circle of radius r.

When constructing the perturbation theory for local fields one encounters the problem of their UV renormalization. To differ unrenormalized fields from renormalized ones, in this section we denote the perturbative CFT fields as $\tilde{A}_l$ to save the notation A_l for their renormalized counterparts. For simplicity we suppose also that $\Delta < 1/2$ to get rid of renormalizations in the interaction hamiltonian and consider the structure functions in the r.h.s. of eq. (1.4) with fields Φ of low enough dimension, so that they require no renormalization.

Consider first the following matrix:

$$\tilde{I}_l^k(g,R,\epsilon) = \langle \tilde{A}^k(\infty)\tilde{A}_l(0)\rangle_g^{(R,\epsilon)}$$
$$= \sum_{n=0}^{\infty} \frac{(-g)^n}{n!}\int_{R>|y_i|>\epsilon} \langle \tilde{A}^k(\infty)\varphi(y_1)\dots\varphi(y_n)\tilde{A}_l(0)\rangle_{\mathrm{CFT}}\,\mathrm{d}^2y_1\dots\mathrm{d}^2y_n\,, \tag{3.1}$$

where the CFT field $\tilde{A}^k$ is placed to infinity. This insertion plays the role of external boundary condition. In eq. (3.1) all the integrations are restricted between the UV cut-off ϵ and the IR cut-off R. Therefore under the suppositions listed above all the integrals are finite and $\tilde{I}_l^k(g,R,\epsilon)$ are regular in the coupling constant g. With the standard CFT normalization of fields,

$$\tilde{I}_l^k(g,R,\epsilon) = \delta_l^k + \mathrm{O}(g)\,. \tag{3.2}$$

Rotational symmetry of the geometry chosen ensures the matrix $\tilde{I}_l^k(g,R,\epsilon)$ to be diagonal in the spin of the fields. It is therefore possible to consider each spin sector in $\mathscr{A}$ separately. Although it is not very essential, to simplify notation in this section we only treat the spin-zero sector and imply that indices $k,l,\dots$ numerate scalar fields only.

The matrix elements $\tilde{I}_l^k(g,R,\epsilon)$ are typically singular as $\epsilon\to 0$. In the limit this singular dependence can be decoupled,

$$\tilde{I}_l^k(g,R,\epsilon) = \sum_{k'} U_{k'}^k(g,\epsilon)\,I_l^{k'}(g,R)\,, \tag{3.3}$$

where $I_l^k(g,R)$ are renormalized matrix elements and $U_l^k(g,\epsilon)$ is the UV cut-off dependent renormalization matrix. Both are regular expansions in the coupling g.

From dimensional arguments

$$U_l^k(g,\epsilon) = \sum_{n=0}^{\infty}\frac{U_l^{k(n)}\left(g\epsilon^{2-2\Delta}\right)^n}{\epsilon^{2(\Delta_l-\Delta_k)}}\,. \tag{3.}$$

As the limit $\epsilon\to 0$ is implied, in the series (3.4) we should only keep terms wi negative powers of ϵ. Since we suppose $\Delta < 1$ this means that there is only a fini number of terms in each matrix element. Moreover, it is clear that if the fields are arranged in order of increasing dimension $\Delta_0 \leqslant \Delta_1 \leqslant \Delta_2 \dots$ then U_l^k h triangular form, i.e.

$$U_l^k(g,\epsilon) = 0 \quad \text{if } \Delta_k > \Delta_l\,. \tag{3.}$$

Obviously the inverse matrix $(U^{-1})_l^k$ has the same properties (3.4) and (3. Define the renormalized perturbative fields as

$$A_k = \sum_l (U^{-1})_k^l \tilde{A}_l\,. \tag{3.}$$

It is natural to keep the following normalization:

$$U_l^k(g,\epsilon) = \delta_l^k + \mathrm{O}(g)\,. \tag{3.}$$

This means that every renormalized field has the form

$$A_l = \tilde{A}_l + \dots\,, \tag{3.8}$$

where a finite number of terms with operators of lower than Δ_l dimension ar omitted in the r.h.s. of eq. (3.8).

The situation becomes somewhat more complicated if any two dimensions diffe in integer number of "quanta" $1-\Delta$, say $\Delta_k - \Delta_l = n(1-\Delta)$, with some positiv integer n(nth order resonance condition). In this case logarithmic divergence show up in perturbation theory and the splitting in eq. (3.3) becomes ambiguous depending on an arbitrary normalization point. In the first-order calculations o this section we shall not encounter any difficulties of this kind.

The renormalized matrix elements

$$I_l^k(g,R) = \langle \tilde{A}^k(\infty)A_l(0)\rangle_g^{(R)} \tag{3.9}$$

are independent on the UV regularization ime dimensional structure as (3.4),

$$I_l^k(g,R) = \sum_{n=0}^{\infty} \frac{I_l^{k(n)}(gR^{2-2\Delta})^n}{R^{2(\Delta_l-\Delta_k)}}, \tag{3.10}$$

ut now we must keep all the terms with positive powers of R. Summation of the eries (3.10) leads to nontrivial functions of R. For general arguments, however, ey behave in a sense homogeneously. This means in particular that the limits

$$\lim_{R\to\infty} \frac{I_l^k(g,R)}{I_0^k(g,R)} = g^{\Delta_k/(1-\Delta)} Q_l^{(k)} \tag{3.11}$$

xist and in general are nothing but the outvac components (1.6). (Of course, oundary conditions at infinity can always be tuned so as to exclude the outvac ontribution. In this case they are components of another out-vector.)

Define also the following set of quantities:

$$G_{\Phi\Phi}^k(g,x,R) = \langle \bar{A}^k(\infty)\Phi(x)\Phi(0)\rangle_g^{(R)}$$

$$= \sum_{n=0}^{\infty} \frac{(-g)^n}{n!} \int_{R>|y_i|} \langle \bar{A}^k(\infty)\varphi(y_1)\ldots\varphi(y_n)\Phi(x)\Phi(0)\rangle_{\mathrm{CFT}}\, \mathrm{d}^2y_1\ldots\mathrm{d}^2y_n .$$

s we supposed no renormalizations of Φ, all the integrals are UV convergent and e need no ϵ. Substituting eq. (1.4) for the product $\Phi(x)\Phi(0)$ we obtain formally

$$C_{\Phi\Phi}^k(x) = \sum_l G_{\Phi\Phi}^l(g,x,R)(I^{-1})_l^k(g,R). \tag{3.13}$$

Ve expect that the expression in the r.h.s. is IR finite, i.e. the limit $R\to\infty$ exists nd provides a set of finite structure functions for the renormalized fields A_k.

Consider the first perturbative order. Let $\mathscr{C}_{\Phi\Phi}^k$, $\mathscr{C}_{\varphi\Phi}^k$ and $\mathscr{C}_{\varphi l}^k$ be the CFT tructure constants,

$$(x\bar{x})^{\Delta_k-2\Delta_\Phi}\mathscr{C}_{\Phi\Phi}^k = \langle \bar{A}^k(\infty)\Phi(x)\Phi(0)\rangle_{\mathrm{CFT}},$$

$$(x\bar{x})^{\Delta_k-\Delta_\Phi-\Delta}\mathscr{C}_{\varphi\Phi}^k = \langle \bar{A}^k(\infty)\varphi(x)\Phi(0)\rangle_{\mathrm{CFT}},$$

$$(x\bar{x})^{\Delta_k-\Delta_l-\Delta}\mathscr{C}_{\varphi l}^k = \langle \bar{A}^k(\infty)\varphi(x)\bar{A}_l(0)\rangle_{\mathrm{CFT}}. \tag{3.14}$$

With these notations one finds to first order

$$\bar{I}_l^k(g,R,\epsilon) = \delta_l^k - g\pi\mathscr{C}_{\varphi l}^k \frac{R^{2(\Delta_k-\Delta_l-\Delta+1)} - \epsilon^{2(\Delta_k-\Delta_l-\Delta+1)}}{\Delta_k-\Delta_l-\Delta+1}. \tag{3.15}$$

Therefore to this order

$$I_l^k(g,R) = \delta_l^k - \frac{g\pi\mathscr{C}_{\varphi l}^k R^{2(\Delta_k-\Delta_l-\Delta+1)}}{\Delta_k-\Delta_l-\Delta+1},$$

$$U_l^k(g,\epsilon) = \delta_l^k + \frac{g\pi\mathscr{C}_{\varphi l}^k \epsilon^{2(\Delta_k-\Delta_l-\Delta+1)}}{\Delta_k-\Delta_l-\Delta+1}. \tag{3.16}$$

To first order in g the structure function (3.13) becomes

$$C_{\Phi\Phi}^k(x) = \mathscr{C}_{\Phi\Phi}^k(x\bar{x})^{\Delta_k-2\Delta_\Phi} - g\int_{R>|y|} \langle \bar{A}^k(\infty)\varphi(y)\Phi(x)\Phi(0)\rangle_{\mathrm{CFT}}\,\mathrm{d}^2y$$

$$+ g\pi\sum_l \frac{\mathscr{C}_{\Phi\Phi}^l\mathscr{C}_{\varphi l}^k R^{2(\Delta_k-\Delta_l-\Delta+1)}}{\Delta_k-\Delta_l-\Delta+1}. \tag{3.17}$$

Eq. (3.17) implies that there is no first-order resonance between Δ_k and any Δ_l. It is clear however how to handle the resonance situation.

Substituting OPE (1.4) into the correlation function in the second term of the r.h.s. of eq. (3.17) (this is allowed if $|y|>r$), one readily observes that the effect of the last term is just to cancel all possible IR divergences. In the absence of first-order resonances we find

$$C_{\Phi\Phi}^k(x) = r^{2(\Delta_k-2\Delta_\Phi)}\Bigg(\mathscr{C}_{\Phi\Phi}^k + g\pi r^{2-2\Delta} \times \sum_l \left[\frac{\mathscr{C}_{\Phi\Phi}^l\mathscr{C}_{\varphi l}^k}{\Delta_k-\Delta_l-\Delta+1} - \frac{\mathscr{C}_{\varphi\Phi}^l\mathscr{C}_{\Phi l}^k}{\Delta_l-\Delta_\Phi-\Delta+1}\right] + \mathrm{O}(g^2)\Bigg). \tag{3.18}$$

Under our conventions about the dimensions of the fields φ and Φ the series in the r.h.s. is convergent. Otherwise it may diverge and require regularization.

The series in eq. (3.18) is not very useful for practical calculations. It is more convenient to evaluate the first-order correction to the structure function $C_{\Phi\Phi}^k(x)$

as an integral,

$$C_{\Phi\Phi}^{k(1)}(r) = -g \int' \langle \bar{A}^k(\infty)\varphi(y)\Phi(x)\Phi(0)\rangle_{\mathrm{CFT}}\, \mathrm{d}^2 y\,, \tag{3.19}$$

throwing away a finite number of IR divergences. This prescription is indicated by the prime on the integral symbol. In the absence of logarithmic divergences it is equivalent to treat the integral (3.19) as an analytic continuation in field dimensions. In the first-order SLYM calculations of this section we shall not meet any logarithms, so this recipe is suitable.

Turn to SLYM. We have

$$C_{\varphi\varphi}^{I(1)}(r) = -ih \int' \langle \varphi(y)\varphi(x)\varphi(0)\rangle_{\mathrm{CFT}}\, \mathrm{d}^2 y\,. \tag{3.20}$$

This integral can be calculated exactly,

$$C_{\varphi\varphi}^{I(1)}(r) = \kappa h r^{16/5} \frac{\pi\gamma^2(1/5)}{14^2\gamma(2/5)}$$

$$= 0.167324465\ldots \times \kappa h r^{16/5}\,, \tag{3.21}$$

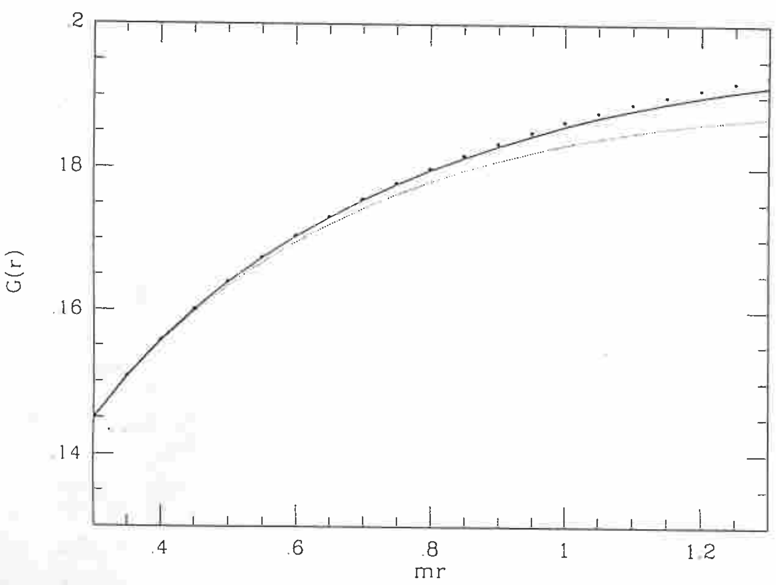

Fig. 2. Effect of the first-order corrections to structure functions. Dotted line: zero-order short-distance expansion. Full line: first-order corrected one. Full circles: large-distance data.

or, using the relation (1.11),

$$C_{\varphi\varphi}^{I} = r^{4/5}\left(1 + 0.0310370062\ldots \times (mr)^{12/5} + \mathrm{O}(r^{24/5})\right). \tag{3.22}$$

The second correction we need is

$$C_{\varphi\varphi}^{\varphi(1)} = -ih \int' \langle \varphi(\infty)\varphi(y)\varphi(x)\varphi(0)\rangle_{\mathrm{CFT}}\, \mathrm{d}^2 y\,. \tag{3.23}$$

TABLE I
Comparison of short- and large-distance expansions and combined data for the correlation function $G(r)/m^4$

	Large-distance expansion		Short-distance expansion		
mr	0-1-2 particles	0-1-2-3 particles	Zeroth-order terms	First-order correction	Combined data
3.000	0.2040233	0.2040233	0.1697161	0.1758804	0.2040233
2.500	0.2027576	0.2027576	0.1788572	0.1884556	0.2027576
2.000	0.2003961	0.2003961	0.1853433	0.1940464	0.2003961
1.800	0.1989299	0.1989299	0.1869155	0.1945682	0.1989299
1.600	0.1970086	0.1970086	0.1877183	0.1941230	0.1970086
1.400	0.1944661	0.1944661	0.1875673	0.1926431	0.1944661
1.200	0.1910576	0.1910576	0.1862015	0.1899715	0.1910576
1.000	0.1864078	1.1864077	0.1832322	0.1858092	1.186408
0.900	0.1834404	0.1834403	1.1809665	0.1830129	0.183440
0.800	0.1799050	0.1799048	0.1780400	0.1796096	0.179905
0.700	0.1756495	0.1756491	0.1743008	0.1754536	0.175649
0.600	0.1704590	0.1704582	0.1695355	0.1703357	0.170458
0.500	0.1640154	0.1640135	0.1634279	0.1639424	0.16401
0.400	0.1558148	0.1558101	0.1554771	0.1557732	0.15581
0.300	0.1449798	0.1449668	0.1448076	0.1449507	0.14497
0.280	0.1423682	0.1423519	0.1422188	0.1423387	0.14235
0.260	0.1395661	0.1395455	0.1394357	0.1395348	0.13955
0.240	0.1365479	0.1365217	0.1364325	0.1365132	0.13652
0.220	0.1332825	0.1332486	0.1331776	0.1332420	0.13325
0.200	0.1297308	0.1296865	0.1296313	0.1296815	0.12969
0.180	0.1258437	0.1257848	0.1257430	0.1257812	0.12579
0.160	0.1215570	0.1214773	0.1214468	0.1214748	0.12148
0.140	0.1167853	0.1166749	0.1166537	0.1166734	0.11668
0.120	0.1114102	0.1112529	0.1112392	0.1112523	0.11125
0.100	0.1052609	0.1050281	0.1050205	0.1050285	0.10503
0.080	0.0980749	0.0977121	0.0977096	0.0977140	0.09771
0.060	0.0894156	0.0888049	0.0888080	0.0888101	0.08881
0.040	0.0784624	0.0772925	0.0773067	0.0773074	0.07731
0.020	0.0633631	0.0603923	0.0604579	0.0604581	0.060458
0.010	0.0530335	0.0466724	0.0468928	0.0468929	0.046893
0.005	0.0475695	0.0355529	0.0361551	0.0361552	0.0361552
0.002	0.0478688	0.0236712	0.0254715	0.0254715	0.0254715
0.001	0.0537990	0.0158697	0.0194741	0.0194741	0.0194741

With the explicit expression for the CFT four-point correlation function [4,5,26] it becomes

$$C^{\varphi(1)}_{\varphi\varphi} = -ihr^{14/5}\int' \mathscr{G}(z,\bar{z})\,\mathrm{d}^2 z\,, \tag{3.24}$$

where

$$\mathscr{G}(z,\bar{z}) = (z\bar{z})^{2/5}[(1-z)(1-\bar{z})]^{1/5}F(\tfrac{2}{5},\tfrac{3}{5},\tfrac{6}{5},z)F(\tfrac{2}{5},\tfrac{3}{5},\tfrac{6}{5},\bar{z})$$
$$-\kappa^2(z\bar{z})^{1/5}[(1-z)(1-\bar{z})]^{1/5}F(\tfrac{1}{5},\tfrac{2}{5},\tfrac{4}{5},z)F(\tfrac{1}{5},\tfrac{2}{5},\tfrac{4}{5},\bar{z})\,. \tag{3.25}$$

The integral was calculated numerically,

$$C^{\varphi(1)}_{\varphi\varphi}(r) = 0.133084702\ldots \times i\pi h r^{14/5}\,. \tag{3.26}$$

Therefore

$$C^{\varphi}_{\varphi\varphi}(r) = \mathscr{C}_{\varphi\varphi\varphi} r^{2/5}\left(1 + 0.021229262\ldots \times (mr)^{12/5} + \mathrm{O}(r^{24/5})\right). \tag{3.27}$$

Substituting the corrected structure functions into the expansion (2.7) we obtain an estimation of the function $G(r)$ up to order $o(r^{24/5})$. The corrected curve is plotted in fig. 1. One can observe in figs. 1 and 2 that in the interval $0.4 < mr < 1.3$ the correction significantly improves the agreement with the large-distance data (see also table 1). The latter are considered in sect. 4.

4. Exact form factors in SLYM; large-distance expansion

In ref. [38] it was proposed to study numerically the correlation functions in 2D integrable models starting from exact form factors of local fields. Consider for simplicity a massive RFT with only one species of massive particles in the asymptotic scattering space (this is just the case in SLYM). The asymptotic states are specified by the number of particles n and a set of their rapidities $\beta_1, \beta_2, \ldots, \beta_n$. The last are convenient to parameterize the on-mass-shell two-momenta (ϵ_i, p_i) of the particles,

$$\epsilon_i = m\cosh\beta_i\,, \qquad p_i = m\sinh\beta_i\,, \tag{4.1}$$

where m is their mass. Denoting the corresponding in- and out-states as $|\beta_1,\ldots,\beta_n\rangle_{\mathrm{in(out)}}$, one can write down the following representation for any two-point correlation function of local fields Φ:

$$\langle\Phi(x)\Phi(0)\rangle = \sum_n \int \frac{\mathrm{d}\beta_1\ldots\mathrm{d}\beta_n}{n!(2\pi)^n}\langle \mathrm{vac}|\Phi(x)|\beta_1,\ldots,\beta_n\rangle_{\mathrm{in}}\,{}^{\mathrm{in}}\langle\beta_1,\ldots,\beta_n|\Phi(0)|\mathrm{vac} \qquad (4$$

The euclidean version of RFT is implied here (imaginary time x^0). Also suppose the following invariant normalization of one-particle states:

$$\langle\beta|\beta'\rangle = 2\pi\delta(\beta-\beta')\,. \qquad (4$$

We can always apply a Lorentz rotation to set $x^1 = 0$, $x^0 = (x\bar{x})^{1/2} = r$. For t geometry eq. (4.2) reads

$$\langle\Phi(x)\Phi(0)\rangle = \sum_n \int \frac{\mathrm{d}\beta_1\ldots\mathrm{d}\beta_n}{n!(2\pi)^n}\langle \mathrm{vac}|\Phi(0)|\beta_1,\ldots,\beta_n\rangle_{\mathrm{in}}\,{}^{\mathrm{in}}\langle\beta_1,\ldots,\beta_n|\Phi(0)|\mathrm{va}$$
$$\times\exp\left[-mr\sum_{i=1}^{n}\cosh\beta_i\right]. \qquad (4$$

Therefore the n-particle term in this expression behaves as $e^{-n(mr)}$ up tc power-like factor and for r large the correlation function is saturated by the low number of particle terms. We expect this series to be well convergent at least large distances r. For a simple example of spin–spin Ising correlation function t convergence was tested numerically in ref. [38]. The same program is applied he to the two-point Θ-correlation in SLYM.

In integrable RFT matrix elements $\langle \mathrm{vac}|\Phi(0)|\beta_1,\ldots,\beta_n\rangle_{\mathrm{in}}$ (called the fo factors) can in many cases be constructed exactly (see refs. [39–41] for fi considerations). Here $\langle\mathrm{vac}|$ is the vacuum state (without particles) of scatteri theory. In a series of papers [28–37] a system of bootstrap conditions for multiparticle form factors was proposed, which provides a systematic way of the reconstruction starting from factorized scattering data. We apply this system recover the form factors

$$F_n(\beta_1,\ldots,\beta_n) = \langle\mathrm{vac}|\Theta(0)|\beta_1,\ldots,\beta_n\rangle_{\mathrm{in}} \qquad (4.$$

in SLYM. The functions $F_n(\beta_1,\ldots,\beta_n)$ are meromorphic functions in each variab β_i. They become "physical" matrix elements (4.5) if all β_i's are real and ordered $\beta_1 > \beta_2 > \ldots > \beta_n$. For opposite ordering $\beta_1 < \beta_2 < \ldots < \beta_n$ they are conjugat matrix elements ${}^{\mathrm{in}}\langle\beta_1,\ldots,\beta_n|\Theta(0)|\mathrm{vac}\rangle$. The bootstrap conditions for this ca read as follows:

(i) $F_n(\beta_1,\ldots,\beta_i,\beta_{i+1},\ldots,\beta_n)=S(\beta_i-\beta_{i+1})F_n(\beta_1,\ldots,\beta_{i+1},\beta_i,\ldots,\beta_n)$, (4.6)

where $S(\beta)$ is two-particle amplitude (1.9).

(ii) $$F_n(\beta_1+\Lambda,\ldots,\beta_n+\Lambda)=F_n(\beta_1,\ldots,\beta_n) \tag{4.7}$$

(iii) $$F_n(\beta_1+2\pi i,\beta_2,\ldots,\beta_n)=F_n(\beta_2,\ldots,\beta_n,\beta_1) \tag{4.8}$$

(iv) As a function of relative rapidities $\beta_{ij}=\beta_i-\beta_j$, with $i<j$, the function $F_n(\beta_1,\ldots,\beta_n)$ exhibits only simple poles in the strip $0\leqslant \operatorname{Im}\beta_{ij}\leqslant\pi$. These are located at $\beta_{ij}=i\pi$ (kinematic poles) and $\beta_{ij}=2i\pi/3$ (bound-state poles). The corresponding residues are

$$-i\operatorname{res}_{\beta'=\beta}F_{n+2}(\beta'+i\pi,\beta,\beta_1,\ldots,\beta_n)=\left(1-\prod_{i=1}^{n}S(\beta-\beta_i)\right)F_n(\beta_1,\ldots,\beta_n) \tag{4.9}$$

and

$$-i\operatorname{res}_{\beta'=\beta}F_{n+1}(\beta'+\tfrac{1}{3}i\pi,\beta-\tfrac{1}{3}i\pi,\beta_1,\ldots,\beta_{n-1})=\Gamma F_n(\beta,\beta_1,\ldots,\beta_{n-1}), \tag{4.10}$$

where

$$\Gamma=i2^{1/2}3^{1/4} \tag{4.11}$$

is the on-mass-shell three-particle vertex. Up to a sign it can be recovered from the scattering amplitude (1.9),

$$-i\operatorname{res}_{\beta=2i\pi/3}S(\beta)=\Gamma^2. \tag{4.12}$$

The vertex is imaginary in SLYM due to the wrong sign in the bound-state pole.

The last two conditions are specific for the operator Θ. They permit to find the overall normalization of the functions $F_n(\beta_1,\ldots,\beta_n)$ and presumably to reconstruct all of them unambiguously.

(v) Interpretation of Θ as a stress tensor component requires

$$F_2(\beta+i\pi,\beta)=\tfrac{1}{2}\pi m^2. \tag{4.13}$$

(vi) The stress tensor conservation

$$\bar\partial T+\partial\Theta=0,\qquad \partial\bar T+\bar\partial\Theta=0, \tag{4.14}$$

where T and $\bar T$ are the left- and right-stress tensor components, allows one to state

$$\langle\text{vac}|T(0)|\beta_1,\ldots,\beta_n\rangle\sum_{i=1}^{n}\mathrm{e}^{\beta_i}=F_n(\beta_1,\ldots,\beta_n)\sum_{i=1}^{n}\mathrm{e}^{-\beta_i},$$

$$\langle\text{vac}|\bar T(0)|\beta_1,\ldots,\beta_n\rangle\sum_{i=1}^{n}\mathrm{e}^{-\beta_i}=F_n(\beta_1,\ldots,\beta_n)\sum_{i=1}^{n}\mathrm{e}^{\beta_i}. \tag{4.15}$$

The T and $\bar T$ form factors defined here have the same singularity structure as the Θ form factors $F_n(\beta_1,\ldots,\beta_n)$. This means that for $n>2$ every function $F_n(\beta_1,\ldots,\beta_n)$ is dividable by the corresponding invariant total energy–momentum without acquiring additional singularities.

To reconstruct the whole set of form factors F_n we start with the two-particle one. Consider the following system of functional equations:

$$f(\beta)=f(2i\pi-\beta),\qquad f(\beta)=S(\beta)f(-\beta). \tag{4.16}$$

The relevant meromorphic solution exhibits a single pole in the strip $0\leqslant\operatorname{Im}\beta<\pi$ at $\beta=2i\pi/3$ and a single zero there at $\beta=0$,

$$f(\beta)=\frac{\cosh\beta-1}{\cosh\beta+1/2}v(i\pi-\beta)v(-i\pi+\beta), \tag{4.17}$$

where $v(\beta)$ is free of poles and zeroes in the half-plane $\operatorname{Im}\beta>0$ and can be defined as the following infinite product:

$$v(\beta)=\prod_{n=1}^{\infty}\left[\frac{(\beta/2i\pi+n+\frac{1}{2})(\beta/2i\pi+n-\frac{1}{6})(\beta/2i\pi+n-\frac{1}{3})}{(\beta/2i\pi+n-\frac{1}{2})(\beta/2i\pi+n+\frac{1}{6})(\beta/2i\pi+n+\frac{1}{3})}\right]^n. \tag{4.18}$$

An integral representation is also possible,

$$v(\beta)=\exp\left(2\int_0^\infty\frac{\mathrm{d}t\sinh\frac{1}{2}t\sinh\frac{1}{3}t\sinh\frac{1}{6}t}{t\sinh^2 t}\mathrm{e}^{i\beta t/\pi}\right). \tag{4.19}$$

For numerical calculations (see below) the following mixed representation is most convenient:

$$v(\beta)=\prod_{n=1}^{N}\left[\frac{(\beta/2i\pi+n+\frac{1}{2})(\beta/2i\pi+n-\frac{1}{6})(\beta/2i\pi+n-\frac{1}{3})}{(\beta/2i\pi+n-\frac{1}{2})(\beta/2i\pi+n+\frac{1}{6})(\beta/2i\pi+n+\frac{1}{3})}\right]^n$$
$$\times\exp\left(2\int_0^\infty\frac{\mathrm{d}t\sinh\frac{1}{2}t\sinh\frac{1}{3}t\sinh\frac{1}{6}t}{t\sinh^2 t}(N+1-N\mathrm{e}^{-2t})\mathrm{e}^{-2Nt+i\beta t/\pi}\right). \tag{4.20}$$

The integral here may be expanded in $1/N$ up to the required order. The function $f(\beta)$ satisfies important relations,

$$f(\beta)f(\beta+i\pi)=\frac{\sinh\beta}{\sinh\beta-i\sin\frac{1}{3}\pi}\,,$$

$$f(\beta+\tfrac{1}{3}i\pi)f(\beta-\tfrac{1}{3}i\pi)=\frac{\cosh\beta+1/2}{\cosh\beta+1}f(\beta)\,. \tag{4.21}$$

Taking into account the normalization condition (4.13) we state

$$F_2(\beta_1,\beta_2)=\frac{\pi m^2}{2}\frac{f(\beta_1-\beta_2)}{f(i\pi)}\,. \tag{4.22}$$

Note that $F_2(\beta_1,\beta_2)$ tends to a finite limit as β_1 (or β_2) $\to\infty$.

Substituting the multiparticle function F_n in the form

$$F_n(\beta_1,\dots,\beta_n)=\prod_{i<j}\frac{f(\beta_i-\beta_j)}{\cosh\frac{1}{2}(\beta_i-\beta_j)}R_n(\beta_1,\dots,\beta_n)\,, \tag{4.23}$$

one readily observes that $R_n(\beta_1,\dots,\beta_n)$ are symmetric and $2i\pi$-periodic in all β_i's entire functions. Under the requirement

$$F_n(\beta_1\dots\beta_n)=\mathrm{O}(1)\quad\text{as } \beta_i\to\infty \tag{4.24}$$

(this seems natural for the form factors of the most "elementary" scalar field Θ) the multiparticle form factors F_n can be represented as

$$F_n(\beta_1,\dots,\beta_n)=H_n\prod_{i<j}\frac{f(\beta_i-\beta_j)}{x_i+x_j}Q_n(x_1,\dots,x_n)\,, \tag{4.25}$$

where $x_i=e^{\beta_i}$, $i=1,\dots,n$,

$$H_n=-\frac{\pi m^2}{4\sqrt{3}}\left(\frac{3^{1/4}}{2^{1/2}v(0)}\right)^n \tag{4.26}$$

and $Q_n(x_1,\dots,x_n)$ are symmetric polynomials in all n variables x of total degree $n(n-1)/2$ and of degree $n-1$ in each variable x_i.

It follows from the residue conditions (4.9) and (4.10) that the polynomials $Q_n(x_1,\dots,x_n)$ satisfy two recurrence relations. The first comes from eq. (4.9),

$$(-)^nQ_{n+2}(x_1,\dots,x_n,x,-x)=x^2Q_n(x_1,\dots,x_n)U_n(x|x_1,\dots,x_n)\,, \tag{4.27}$$

where (with the notation $\omega=e^{i\pi/3}$)

$$U_n(x|x_1,\dots,x_n)=\frac{1}{2x(\omega-\omega^{-1})}\left[\prod_{i=1}^n(x+\omega x_i)(x-\omega^{-1}x_i)-\prod_{i=1}^n(x-\omega x_i)(x+\omega^{-1}x_i)\right]$$

$$=\sum_{k=0}^{n-1}x^{2n-2k-2}\sum_{l=0}^{k}\frac{\sin(2k+l+1)\frac{1}{3}\pi}{\sin\frac{1}{3}\pi}\sigma_l\sigma_{2k+1-l}\,, \tag{4.28}$$

where $\sigma_l(x_1,\dots,x_n)$ are elementary symmetric polynomials,

$$\prod_{i=1}^n(x+x_i)=\sum_k x^{n-k}\sigma_k(x_1,\dots,x_n)\,. \tag{4.29}$$

The second is a consequence of the bound-state residue (4.10),

$$\frac{Q_{n+2}(x_1,\dots,x_n,\omega x,\omega^{-1}x)}{x\prod_{i=1}^n(x+x_i)}=Q_{n+1}(x_1,\dots,x_n,x)\,. \tag{4.30}$$

One readily finds

$$Q_1=1\,,\qquad Q_2=\sigma(x_1,x_2)\,. \tag{4.31}$$

The stress tensor conservation (4.14) restricts polynomials $Q_n(x_1,\dots,x_n)$ with $n>2$ to have the form

$$Q_n(x_1,\dots,x_n)=\sigma_1(x_1,\dots,x_n)\sigma_{n-1}(x_1,\dots,x_n)P_n(x_1,\dots,x_n)\,, \tag{4.32}$$

where $P_n(x_1,\dots,x_n)$ are again symmetric polynomials of total degree $n(n-3)/$ and of degree $n-3$ in each variable. The recurrence relations (4.28) and (4.30) become

$$(-)^{n+1}P_{n+2}(x_1,\dots,x_n,x,-x)=U_n(x|x_1,\dots,x_n)P_n(x_1,\dots,x_n)\,, \tag{4.33}$$

$$P_{n+2}(x_1,\dots,x_n,\omega x,\omega^{-1}x)=\prod_{i=1}^n(x+x_i)P_{n+1}(x_1,\dots,x_n,x)\,. \tag{4.34}$$

From these it is easy to find

$$P_3 = 1\,,$$

$$P_4 = \sigma_2(x_1, \ldots, x_4)\,,$$

$$P_5 = \sigma_2(x_1, \ldots, x_5)\sigma_3(x_1, \ldots, x_5) - \sigma_5(x_1, \ldots, x_5)\,. \tag{4.35}$$

The system of functional relations (4.33) and (4.34) turns out to be compatible and seems to have a unique solution. This was checked up to P_8. It can also be verified that the following general expression:

$$P_n(x_1, \ldots, x_n) = \det \Sigma\,, \tag{4.36}$$

where Σ is $(n-3)\times(n-3)$ matrix with the entries

$$\Sigma_{ij} = \sigma_{3i-2j+1}(x_1, \ldots, x_n)\,, \tag{4.37}$$

i.e.

$$\Sigma = \begin{pmatrix} \sigma_2 & \sigma_5 & \sigma_8 & \sigma_{11} & \ldots \\ 1 & \sigma_3 & \sigma_6 & \sigma_9 & \ldots \\ 0 & \sigma_1 & \sigma_4 & \sigma_7 & \ldots \\ 0 & 0 & \sigma_2 & \sigma_5 & \ldots \\ & & \ldots & & \end{pmatrix}$$

for $n > 3$ provides a solution to the system (4.33) and (4.34). It seems natural to consider this solution as a complete set of exact Θ form factors in SLYM. It is interesting to note that substituting formally $n = 0$ and $Q_0 = 1$ in eq. (4.25) one recovers exactly the vacuum expectation value (1.10),

$$F_0 = \langle \Theta \rangle\,. \tag{4.38}$$

In terms of the form factors found the two-point Θ correlation reads, according to eq. (4.4),

$|F_n|^2$

$$G(r) = \sum_{n=0}^{\infty} \int \frac{d\beta_1 \ldots d\beta_n}{n!(2\pi)^n} F_n(\beta_1, \ldots, \beta_n) F_n(\beta_n, \ldots, \beta_1) \exp\left(-mr \sum_{i=1}^{n} \cosh \beta_i\right). \tag{4.39}$$

The purely imaginary three-particle vertex (4.11) manifests itself in negative contributions of the odd number of particle terms. This results in alternating series and causes very fast convergence. The first four terms (i.e. zero-, one-, two- and three-particle contributions) were calculated numerically, using the following ex-

plicit expressions for the corresponding form factors:

$$F_0 = -\frac{\pi m^2}{4\sqrt{3}}\,,$$

$$F_1 = -\frac{i\pi m^2}{2^{5/2} 3^{1/4} \upsilon(0)}\,,$$

$$F_2(\beta_1, \beta_2) = \frac{\pi m^2}{2} \frac{f(\beta_{12})}{4\upsilon^2(0)}\,,$$

$$F_3(\beta_1\beta_2\beta_3) = \frac{i 3^{1/4} \pi m^2}{2^{7/2} \upsilon^3(0)} \prod_{i<j}^{3} f(\beta_{ij}) \left(1 + \frac{1}{8\prod_{i<j} \cosh \frac{1}{2}\beta_{ij}}\right). \tag{4.40}$$

The numerical results are shown in figs. 1–3. One can see extremely fast convergence in the region $mr > 0.001$ (this is illustrated in fig. 3). With only a few terms taken into account we get rather precise data for $mr > 0.01$. Comparing these with the results of the short-distance expansion (see fig. 2), we observe a good match

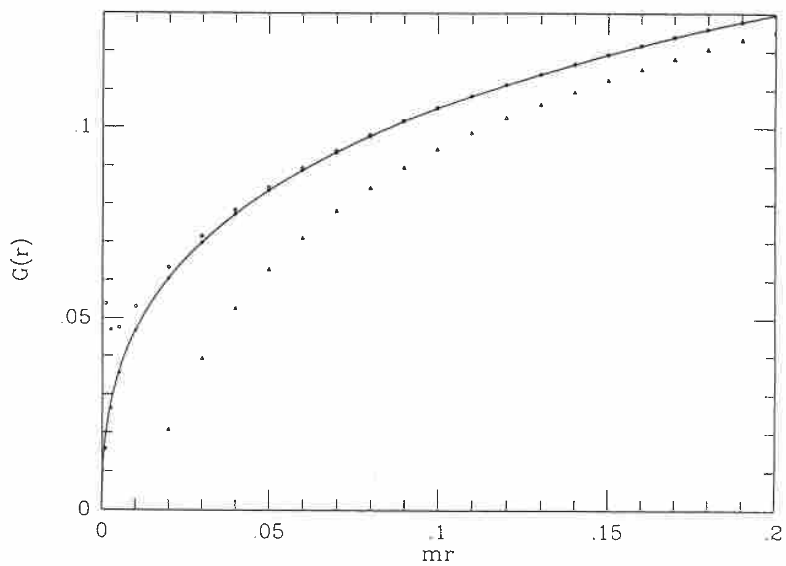

Fig. 3. Convergence of the large-distance expansion for small mr. Empty triangles: zero- and one-particle contributions. Empty circles: the same plus two-particle term. Full circles: up to three-particle state contributions. Full curve: the short-distance data.

inside a sufficiently wide region $0.01 < mr < 1.00$. Sewing together these two sets of data we find the combined estimation for all distances, listed in table 1.

5. Conclusions

(i) When developing perturbation theory around the UV conformal limit it is convenient to accumulate non-analytic contributions into a set of numbers (outvac components). Then analytic corrections are separated into OPE structure functions and can be calculated systematically. The exact calculation of VEVs in integrable RFT remains an open problem.

(ii) If negative dimension operators are present in CFT, the perturbed correlation functions may differ in their UV asymptotic form from naive CFT predictions. Note that negative dimension perturbation not necessarily leads to non-unitary RFT, but may preserve unitarity. This is exactly what happens in the sinh-Gordon model, where CFT of a massless scalar field ϕ is perturbed by the primary operator $\cosh(\beta\phi)$ of negative dimension $\Delta = -\beta^2$. Corresponding massive RFT is obviously unitary and the two-point function $\langle e^{\beta\phi(x)} e^{\beta\phi(0)} \rangle$ behaves as $r^{-4\beta^2}$ in the UV limit (rather than $\sim r^{4\beta^2}$ as predicted by CFT) due to nonzero VEV of the next negative dimension operator $e^{2\beta\phi}$.

(iii) Correlation functions in integrable massive RFT models can be evaluated numerically using corresponding exact form factors. In some cases this may help to understand the off-mass-shell physics.

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